UNIT 6 INDUCTOR AND INDUCTANCE

6.1 Inductor and inductance

6.1.1 Associated quantities

- **Inductor**, also called a choke, is another passive type electrical component designed to take advantage of this relationship by producing a much stronger magnetic field than one that would be produced by a simple coil.

- **Symbol** of inductance is \( L \).

- **Unit** of inductance is **Henry**.

- **Inductance** – the property of an electric circuit by which an electromotive force is induced in it as the result of changing magnetic flux.

- **Electromagnet** – temporary magnet production due to flow of electric current.

- **Electromagnetic induction** - production process electric form magnet.

6.1.2 Types of inductor

<table>
<thead>
<tr>
<th>Fixed</th>
<th>Air core</th>
<th><img src="image1.png" alt="Diagram" /></th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron core</td>
<td><img src="image2.png" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>Ferrite core</td>
<td><img src="image3.png" alt="Diagram" /></td>
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</tbody>
</table>

| Variable | Core loss | ![Diagram](image4.png) |

6.1.3 Construction of inductor

An inductor is usually constructed as a coil of conducting material, typically copper wire, wrapped around a core either of air or ferrous material.

Core materials with higher permeability than air confine the magnetic field closely to the inductor, thereby increasing the inductance. Inductors come in many shapes. Most are constructed as enamel coated wire wrapped around a ferrite with wire exposed on the outside, while some enclose the wire completely in ferrite and are called ‘shielded’. 
Some inductors have an adjustable core, which enables changing of the inductance. Small inductors can be etched directly onto a printed board by laying out the trace in a spiral pattern.

6.2 Inductance equivalents circuit for series and parallel connection

6.2.1 Inductors connected in series

![Inductor in series](image1)

Total voltage,
\[ E_T = e_1 + e_2 \]
Total inductance,
\[ L_T = L_1 + L_2 \]
It follows that for \( n \) series connected inductors
\[ L_T = L_1 + L_2 + \ldots + L_n \]
Current, \( I = I_{L1} = I_{L2} \)

6.2.2 Inductors connected in parallel

![Inductor in parallel](image2)

Total voltage,
\[ E_T = e_1 = e_2 \]
Total current,
\[ I_T = I_1 + I_2 \]
Total inductance
\[ \frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} \]
It follows that for \( n \) parallel connected inductors
\[ \frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n} \]
6.2.3 Inductors connected in series-parallel

![Inductor in series-parallel connection](image)

Figure 5: Inductor in series-parallel connection

Total current,
\[ I_T = I_1 + I_2 \]
Total inductance,
\[ L_T = L_1 + \frac{L_2 L_3}{L_2 + L_3} \]

**Example 1**

Calculate the equivalent inductance of two inductors of 3H and 5H connected:
(a) in series  
(b) in parallel.

Solution:

\( a) \) Total inductance in series,
\[ L_T = L_1 + L_2 = 3H + 5H = 8H \]

\( b) \) Total inductance in parallel,
\[ L_T = \frac{L_1 L_2}{L_1 + L_2} = \frac{3H \times 5H}{3H + 5H} = 1.875H \]

**Example 2**

Find the inductance to be connected in parallel with a 10H capacitor for the equivalent capacitance to be 6H.

Solution:

\[ L_T = 6H \text{, } L_1 = 10H \text{, } L_2 = \text{unknown capacitance} \]

For two capacitance in series:

\[ \frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} \]
\[ L_2 = \frac{L_1 L_T}{L_1 - L_T} = \frac{10H \times 6H}{10H - 6H} = 15H \]

**Example 3**

Find the total inductance for the circuit below

Total inductance,
\[ L_T = L_1 + \frac{L_2 L_3}{L_2 + L_3} \]
\[ L_T = 5H + \frac{2H \times 3H}{2H + 3H} \]
\[ L_T = 6.2H \]
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Find the total inductance of the circuit.

1. [Ans: 0.788H]

2. [Ans: 8.33mH]

3. [Ans: 3.54H]

4. [Ans: 20mH]
6.3 Circuit with inductive load

6.3.1 Electromagnetic induction

When a conductor is moved across a magnetic field so as to cut through the lines of force (or flux, an electromotive force (e.m.f) is produced in the conductor. If the conductor forms part of a closed circuit then the e.m.f produced causes an electric current to flow round the circuit. Hence an e.m.f is induced in the conductor as a result of its movement across the magnetic field. This effect is known as ‘electromagnetic induction’.

6.3.2 Faraday’s Law

Faraday’s laws of electromagnetic induction state:

i) An induced e.m.f is set up whenever the magnetic field linking that circuit changes

ii) The magnitude of the induced e.m.f in any circuit is proportional to the rate of change of the magnetic flux linking the circuit.

6.3.3 Mathematical relationship between the induced e.m.f and the network

Faraday noted that the e.m.f induced in a loop is proportional to the rate of change of magnetic flux through it:

\[ e = -N \frac{d\Phi}{dt} \]

Where; \( e \) is the electromotive force induced (in volts)
\( N \) is the number of turns of the coil
\( d\Phi \) is the change of flux in Weber, Wb
\( dt \) is the time taken for the flux to change in seconds.

*Notice the negative sign is the induced current will now produce an induced magnetic field. The direction of that magnetic field will be opposite to the direction the flux is changing.

6.3.4 Self-inductance and the induced e.m.f

Inductance is the name given to the property of a circuit whereby there is an e.m.f induced into the circuit by change of flux linkages produced by a current change. When the e.m.f is induced in the same circuit as that in which the current is changing, the property is called self-inductance, \( L \).

Induced e.m.f is the product of self-inductance and the rate of change in current

\[ e = L \frac{di}{dt} \]

Where; \( e \) is induced e.m.f (in volts)
\( L \) is self-inductance in Henry
\( \frac{di}{dt} \) is the change of current in Amperes
\( dt \) is the time taken for the current to change in seconds.
6.3.5 The mathematical of self-inductance

\[ L = \frac{N^2 \mu \mu_r A}{l} \]

Where;  
- \( N \) is number of turns of coil  
- \( L \) is length of coil  
- \( A \) is surface area  
- \( \mu \) is permeability

6.3.6 The factors that influence inductance

A component called an inductor is used when the property of inductance is required in a circuit. The basic form of an inductor is simply a coil of wire.

**Factors which affect the inductance of an inductor include:**

i) **The number of turns of wire** (\( N \)) – more turns the higher the inductance  
ii) **The cross-sectional area of the coil of wire** (\( A \)) – the greater the cross-sectional area the higher the inductance  
iii) **The presence of magnetic core** - when the coil is wound on an iron core, the same current sets up a more concentrated magnetic field and the inductance is increased  
iv) **The way turns are arranged** – a short tick coil of wire has a higher inductance than the along thin one.

6.4 Rise and decay of current goes through an inductor in the dc circuit

6.4.1 Rise and decay of current

Refer to Figure 5, when switch in ‘a’ position, inductor connected to DC supply. The current had not achieved maximum value immediately. The current are going to reach maximum value in a period of time that certain caused by production e.m.f induced by inductor which always against the supply voltage. In other words, the current of the circuit is rise delayed.

When switch is being transformed to position ‘b’, inductor circuit had short circuit (no supply voltage). The current is not decrease continue to zero but take a time that certain from maximum value until zero value. Refer to figure 6 which is shown the exponential graph changing of current in inductor circuit.
6.4.2 Time constant

Time constant, \( \tau \) defines as time for current achieve maximum \( (I_M) \) if this maintain the early promotion rate current.

i) Time constant at rise of current

Practically, the current did not rise by regular. By graphically, it achieves 63.2\% from maximum value (point ‘B’ in figure 7) in time constant. In other words, time constant, \( \tau \) also defines as time for current of inductor achieve 63.2\% from the maximum value.

From the graph:
- Current will be rising from minimum value (0) by exponent headed for maximum value, \( I_M \) (steady state).
- Time for value of \( i \) achieve 63.2\% from maximum value is time constant, \( \tau \).
- Time for value of \( I \) achieve maximum value is \( 5\tau \)

From RL circuit, time constant, \( \tau \), given by equation:

\[
\tau = \frac{L}{R}
\]
ii) Time constant at decay of current

In decay of current through an inductor, a method to find values of time constant same as in rise of current through an inductor. The differences are value of current decay from maximum value \(I_M\) to minimum (0), and value 63.2% replaced with 36.8% which is 100% - 63.2%. Figure 8 shown clear pictures for decay of current in inductor.

From the graph:
- Current will reduce from maximum value \(I_M\) by exponentially until minimum value (0).
- Time for \(i\) to reach 36.8% from maximum value (reducing of 63.2% from origin value, \(I_M\)) is time constant: \(\tau\)
- Time for \(i\) to reach final value (zero) is \(5\tau\).

Figure 8: Graph decay of current through an inductor

6.4.3 Energy stored in an inductor

An inductor possesses an ability to store energy. The energy stored, \(W\) in the magnetic field of an inductor is given by:

\[
E = \frac{1}{2}LI^2
\]

**Example 1**

One inductor 0.5H connected in series with resistor 20Ω and dc voltage 120V. Determine:

i) Time constant

\[
\tau = \frac{L}{R} = \frac{0.5}{20} = 0.025\text{s}
\]

ii) Current at time 0.025s

\[
i = I_M\left(1 - e^{-\frac{Rt}{L}}\right)
\]

\[
i = 6\left(1 - e^{-\frac{20(0.025)}{0.5}}\right)
\]

\[
i = 6\left(1 - e^{-1}\right)
\]

\[
i = 3.79\text{A}
\]

iii) Energy store,

\[
E = \frac{1}{2}LI^2
\]

\[
E = \frac{1}{2} \left(0.5\right)(6^2) = 9\text{ Joule}
\]
Example 2

When switch connected to ‘a’, calculate:

i) Time constant
ii) Time taken for current achieve maximum value
iii) Maximum current if the current is 2.5A in 0.38s.

Solution

i) Time constant,
\[ \tau = \frac{L}{R} = \frac{7.5}{10} = 0.75\text{s} \]

ii) \(5\tau = 5(0.75) = 3.75\text{s}\)

Example 3

One circuit has resistor 40Ω connected in series with inductor 15H and dc voltage 220V. Calculate:

i) Time constant
ii) Current at time (i)
iii) Current at time 0.05s
iv) Energy stored in inductor

Solution

i) Time constant
\[ \tau = \frac{L}{R} = \frac{15}{40} = 0.375\text{s} \]

ii) \(i = I_M(1 - e^{-Rt/L})\)
\[ I_M = \frac{V}{R} = \frac{220}{40} = 5.5\text{A} \]
\[ i = 5.5(1 - e^{-40(0.05)/15}) \]
\[ i = 5.5(1 - e^{-0.133}) \]
\[ i = 5.5(1 - e^{-1}) \]
\[ i = 3.47\text{A} \]

iii) \(t = 0.05\text{s}\)
\[ i = 5.5(1 - e^{-40(0.05)/15}) \]
\[ i = 5.5(1 - e^{-0.133}) \]
\[ i = 5.5\text{A} \]

iv) \(E = \frac{1}{2}LI^2\)
\[ E = \frac{1}{2}(15)(5.5^2) = 226.875\text{ Joule} \]
1. A coil of inductance 0.04 H and resistance 10Ω is connected to a 120 V, d.c. supply. Determine
   (a) the final value of current
   (b) the time constant of the circuit
   (c) the value of current after a time equal to the time constant from the instant the supply
   voltage is connected.

   \[12A, 4\text{ms}, 7.58A\]

2. The winding of an electromagnet has an inductance of 3H and a resistance of 15Ω. When it is
   connected to a 120 V d.c. supply, calculate:
   (a) the steady state value of current flowing in the winding
   (b) the time constant of the circuit
   (c) the value of the induced e.m.f. after 0.1s
   (d) the time for the current to rise to 85% of its final value
   (e) the value of the current after 0.3 s

   \[8A, 0.2s, 72.78V, 0.379s, 6.215A\]

3. A coil has an inductance of 1.2H and a resistance of 40Ω and is connected to a 200 V, d.c. supply.
   Determine the approximate value of the current flowing 60 ms after connecting the coil to the
   supply.

   \[4.3\text{A}\]

4. A 25 V d.c. supply is connected to a coil of inductance 1H and resistance 5Ω. Determine the
   approximate value of the current flowing 100 ms after being connected to the supply.

   \[2\text{A}\]

5. The field winding of a 200 V d.c. machine has a resistance of 20Ω and an inductance of 500mH.
   Calculate:
   (a) the time constant of the field winding
   (b) the value of current flow one time constant after being connected to the supply
   (c) the current flowing 50 ms after the supply has been switched on.

   [(a) 25 ms (b) 6.32 A (c) 8.65 A]